**Barron’s Math 360: A Complete Study Guide to Geometry**

# Chapter 1: Building A Geometry Vocabulary

## The Building Blocks of Geometry

* Undefined Terms. Terms so fundamental they cannot be defined, but they can be described.
* Defined Terms.
* Postulates. A postulate is a statement that is accepted without proof.
* Theorems. A theorem is a generalization that can be proved to be true.

Geometry is an example of a postulational system in which a beginning set of assumptions and undefined terms is used as a starting point in developing new relationships that are expressed as theorems.

### Undefined Terms

* Point. Indicates position, but has no length, width or dept.
* Line. A line is a set of continuous points that extend indefinitely in either direction.
* Plane. A plane is a set of points that forms a flat surface that has no depth and extends indefinitely in all directions.

### Defined Terms

1. Line Segment. A *line segment* is part of a line consisting of two points, called *endpoints*, and the set of all points between them.
2. Ray. A *ray* is part of a line consisting of a given point, called the *endpoint*, and the set of all points on one side of the endpoint.
3. Opposite rays. Opposite rays are rays that have the same endpoint and that form a line.
4. Angle. An *angle* is the union of two rays having the same endpoint. The endpoint is called a *vertex* of the angle, and the rays are called the *sides* of the angle.

### Naming Angles

An angle may be named in one of three ways.

1. Using three letters, the center letter corresponding to the vertex of the angle and the other letters representing points on the sides of the angle.
2. Placing a number at the vertex and the *interior* of the angle.
3. Using a single letter that corresponds to the vertex, provide this does not cause any confusion.

## Definitions and Postulates

### Definitions

The purpose of a definition is to make the meaning of the term clear. A good definition must:

* Clearly identify the word being defined.
* State the distinguishing characteristics.
* Be expressed in a grammatically correct sentence.
* A good definition must be reversible.

### Definitions of Collinear and Non-Collinear Points

* *Collinear points* are points that lie on the same line.
* *Non-collinear points are points that do not lie on the same line.*

### Definition of a Triangle

A *triangle* is a figure formed by connecting three non-collinear points with three different line segments, each of which has two of these points as endpoints.

#### The Reversibility Test

The reverse of a definition must be true.

A *midpoint* of a segment may be defined as a point that divides a segment into two segments of equal length.

A point that divides a segment into two segments of equal length is the midpoint of that segment.

### Initial Postulates

Not everything can be proved. There must be some basic assumptions, called postulates (or axioms), that are needed at the beginning.

1. Two points *determine* a line.

## Inductive Versus Deductive Reasoning

*Inductive reasoning* involves examining a few examples, observing a pattern, and then assuming the pattern will never end. Inductive reasoning is ***not*** a valid form of proof.

*Deductive reasoning* uses accepted *facts* (undefined terms, defined terms, postulates, and previously established theorems) to reason in a step-by-step fashion until a desired conclusion is reached.

## The IF… THEN… Sentence Structure

The statement in the “If” clause identifies the *condition* that must be met.

* *After* a theorem is proved, the “then” clause represents the fact you are allowed to apply whenever the condition in the “if” clause is true.
* *Before* a proposed theorem is proved, the “if” clause contains what we know and the “then” clause identifies what we need to prove.

## Summary

* Geometry is a postulational system built upon undefined terms, defined terms, and postulates, which are used to build theorems through a logical chain of reasoning, either inductive or deductive.
* Undefined terms are point, line, and plane.
* Defined terms in this chapter are line segment, ray, opposite rays, angle, collinear points, and non-collinear points.
* Using correct techniques in naming terms is essential for appropriate mathematical communication.

## Review Exercises

1. For the accompanying diagram:
   1. Name four rays, each of which has point B as an endpoint.
   2. Name line in four different ways. .
   3. Name line in four different ways. .
   4. Name four angles that have the same vertex. .
   5. Name two pairs of opposite rays.
2. Name the vertex of each angle.
3. 1 -
4. 3 -
5. 5 -
6. Use 3 letters to name each angle.
7. 2 -
8. 4 -
9. 6 -
10. Name 4 collinear points. .
11. If point is the midpoint , name two segments that have the same length.
12. Name different triangles that appear in the diagram.

1. Name each angle that has point as its vertex.
2. Name an angle that is not an angle of a triangle.
3. Name two pairs of opposite rays.

1. Name a segment that is a side of two different triangles.
2. To prove is the midpoint of , which two segments must be demonstrated to have the same length?
3. Write the reverse of each of the following definitions.
4. An acute angle is an angle whose measure is less than 90.

An angle whose measure is less than 90 is an acute angle.

1. An equilateral triangle is a triangle having three sides equal in length.

A triangle having three sides equal in length is an equilateral triangle.

1. A bisector of an angle is the ray (or segment) that divides the angle into two congruent angles.

The ray (or segment) that divides the angle into two congruent angles is a bisector of an angle.

1. Identify each of the following as an example of inductive or deductive reasoning.
2. The sum of 1 and 3 is an even number, the sum of 3 and 5 is an even number, the sum of 5 and 7 is an even number, the sum of 7 and 31 is an even number; the sum of 19 and 29 is an even number. Conclusion: The sum of any two odd numbers is an even number.

Inductive reasoning.

1. All students in Mr. Euclid’s geometry class are 15 years old. John is a member of Mr. Euclid’s class. Conclusion: John is 15 years old.

Deductive reasoning.

1. It rained on Monday, Tuesday, Wednesday, Thursday, and Friday.

Inductive reasoning.

1. The sum of measures of a pair of complementary angles is 90. Angle and angle are complementary. The measure of angle is 50. Conclusion: The measure of angle is 40.

Deductive reasoning.

1. A *median* of a triangle is a segment drawn from a vertex of the triangle to the midpoint of the opposite side of the triangle Draw several large right ( triangles. See the diagram. For each triangle, locate the midpoint of the hypotenuse (the side opposite the angle). Draw the median to the hypotenuse. Using a ruler, compare the lengths of the median and the hypotenuse in each triangle being drawn. Use inductive reasoning to draw an appropriate conclusion. Note in the diagram that is the midpoint of measure the same length.
2. Draw several large triangles (not necessarily right triangles). In each triangle locate the midpoint of each side. Draw the three medians of each triangle. Use inductive reasoning to draw a conclusion related to where the medians intersect.
3. Use deductive reasoning to arrive at a conclusion based on the assumptions given.

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| --- | --- |
| ASSUMPTIONS | 1. All martians have green yes.  2. Henry is a martian.  Conclusion: Henry has green eyes. |

|  |  |
| --- | --- |
| ASSUMPTIONS | 1. The sum of the measures of the angles of a triangle is 180.  2. In a particular triangle, the sum of the measures of two angles is 100  Conclusion: The third angle is |

1. A prime number is any whole number that is divisible only by itself and 1. For example, 7, 11, and 13 are prime numbers. Evaluate the formula using all integer values from 0 to 9, inclusive. Do you notice a pattern.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  | 17 |  |
| 0 | 0 | 0 | 17 | 17 |
| 1 | 1 | 1 | 17 | 19 |
| 2 | 4 | 2 | 17 | 23 |
| 3 | 9 | 3 | 17 | 29 |
| 4 | 16 | 4 | 17 | 37 |
| 5 | 25 | 5 | 17 | 47 |
| 6 | 36 | 6 | 17 | 59 |
| 7 | 49 | 7 | 17 | 73 |
| 8 | 64 | 8 | 17 | 89 |
| 9 | 81 | 9 | 17 | 107 |
| 16 | 256 | 16 | 17 | 289 |

Prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107

1. Use inductive reasoning, draw a conclusion.

All numbers generated are prime numbers.

Conclusion: The formula will always generate prime numbers.

1. No, it is not true for .